MODELING GRIDDING ERROR ON LiDAR-DERIVED DIGITAL ELEVATION MODELS

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Abstract:

A hybrid theoretical-empirical model has been developed for modelling the error of LiDAR derived Digital Elevation Models (DEMs) under non-open terrain, especially designed to be applied in forestry applications. The theoretical component seeks to model the propagation of the Sample Data Error (SDE), i.e. error from LiDAR data capture of ground sampled points on open terrain, towards interpolated points. The interpolation methods used for infilling gaps may produce a non-negligible error that is referred to from here onwards as gridding error. In this case, interpolation is performed using an Inverse Distance Weighted (IDW) method with the local support of the five closest neighbours, although it would be possible to utilise other interpolation methods. The empirical component refers to what is known as "Information Loss". That is the error due purely to modelling the continuous terrain surface from only a discrete number of points plus the error arising from the interpolation process. The SDE must be previously calculated from a suitable number of check points located in open terrain and supposing the LiDAR point density is sufficiently high to neglect the gridding error. For model calibration, 29 study sites, 200 x 200 m in size, were acquired by means of stereo-photogrammetric methods belonging to different areas around Almeria province, south-east Spain. The developed methodology was validated against two different LiDAR data sets. The first data set corresponds to an Ordnance Survey (OS) LiDAR survey carried out over a region of Bristol in the UK. The second data set corresponds to an area located at Gador mountain range, south of Almería province, Spain. Both terrain slope and sampling density were incorporated in the empirical component through the calibration phase, resulting in a very good agreement between predicted and observed data (R2 = 0.9856; p<0.001). Regarding the validation results, the Bristol observed vertical errors, corresponding to a different LiDAR point density, offered a reasonably good fit to predicted errors. Similarly promising results were achieved in the more rugged morphology of the Gador mountain range data set. Despite the fact that results to date should be regarded as preliminary, the findings presented in this paper could be used as a guide for the selection of appropriate operational parameters (essentially point density in order to optimize survey cost), in projects related to LiDAR survey in non-open terrain, for instance those projects dealing with forestry applications.

Keywords: LiDAR, digital elevation models, accuracy, spatial modeling, topographic mapping

1. INTRODUCTION

Accurate and high spatial resolution Digital Elevation Models (DEMs) from airborne LiDAR data are in increasing demand for a growing number of mapping and GIS tasks related to applications such as forest management, urban planning, bird population modeling, ice sheet mapping, flood control, road design, etc. [1]. However, many end users of DEMs are unaware of the issues surrounding the quality of the underlying height data and their influence in derived calculations such as slope and aspect [2].

In fact, many variables are known to contribute to the accuracy of LiDAR-derived DEMs. Among them, LiDAR sampling density is considered to be a significant contributor to the vertical error [3], although terrain morphology and land cover (filtering performance to transform a Digital Surface Model (DSM) to a bare-earth Digital Terrain Model (DTM)) have to be quoted as other important factors influencing the final DEM accuracy. But unlike the last two variables, sampling density (or post-spacing) represents a significant portion of overall survey costs [4] that, furthermore, can be selected as an operational parameter of the project.

Although there have been a number of studies dealing with LiDAR-derived DEM error, most can be classified as empirical work in which the influence of different variables on DEM error was analysed ([5], [6], [7]). In this way, a hybrid theoretical-empirical model has been developed for modeling the error of LiDAR-derived DEMs under non-open terrain, especially designed to be applied in forestry applications where the laser beam penetration through canopy can be limited and so the ground sampling density reduced (Figure 1). There is a frequent need to densify the initial LiDAR point cloud (last pulse) when the surveyed area presents dense vegetation [1] and new ground points have to be interpolated to infill gaps and construct accurate DTMs and Canopy Height Models (CHMs). But the interpolation methods used for infilling gaps may produce a non-negligible error that is referred to from here onwards as gridding error [2]. That is the propagation of the Sample Data Error (SDE) towards interpolated points. Obviously, gridding error depends on, among other variables, the interpolation method employed ([8], [9]).

Therefore the goal of this research is to establish a relationship between LiDAR sampling density, terrain morphology, interpolation method and DEM accuracy.

2. MODEL DEVELOPMENT

The full model developed in this work can be expressed as the sum of three components:

$$\sigma^2_{total} = \sigma^2_{SDE} + \sigma^2_{gridding} + \sigma^2_{\text{filtering}} \qquad (1)$$

Where σ^2_{total} is the total vertical error estimated by the model, σ^2_{SDE} is the sample data error, *i.e.* error from LiDAR data capture of ground sampled points on open terrain, $\sigma^2_{gridding}$ is the aforementioned gridding error, and $\sigma^2_{filtering}$ is the error due to filter non-terrain objects (buildings, vegetation, etc.) to obtain the final bare-earth DTM. All the terms are expressed as error variances. Equation 1 is based on general error-propagation theory, assuming that the sources of error are independent or uncorrelated and, likewise, that the errors are randomly distributed.

Regarding Figure 1, filtering error can be neglected in open terrain because of the absence of non-terrain objects. Equally, gridding error can also be excluded in open terrain because of the high point density usually offered by LiDAR technology. Hence the SDE could be previously calculated from a suitable number of check points located in open terrain [10]. Notice that filtering error, non-negligible over non-open terrain, is very cumbersome to model because it depends on the algorithm used to filter the LiDAR data, the type and density of vegetation, terrain complexity and so on. Hence it may be recommended to take it into account by adding the expected filtering error to the SDE in equation 1 as a specific sample data error for every type of land cover distinguished over the entire working area. Hence equation 1 can then be rearranged as:

$$\sigma^2_{total} = \sigma^2_{SDE} + \sigma^2_{gridding} \qquad (2)$$

So the remaining question is how to model gridding error. A hybrid theoretical-empirical approach has been proposed in this work. The theoretical component seeks to model the propagation of the sample data error obtained for every land cover towards interpolated points. In this case, interpolation is performed using the Inverse Distance Weighted (IDW) method using a power of two and with the local support of the five closest neighbours, although it would be possible to utilise other interpolation methods.

A brief explanation of the deduced equation is presented through equations 3 to 6 for the particular case of three closest neighbours.

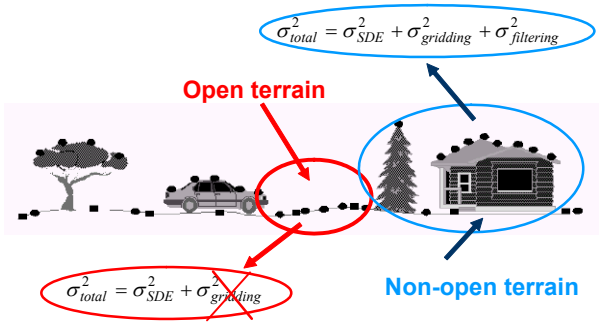

Fig. 1. Schematic illustration depicting the fundamental concepts for the model development.

The variance of Z_0 (interpolated point) and Z_i (three sample points in this case) can be estimated using general error propagation theory, supposing the heights Z as a random variable second-order stationary. If the interpolation weights for every point take the values a, b and c respectively, it can be written:

$$z_0\left(\frac{1}{d_1^2}+\frac{1}{d_2^2}+\frac{1}{d_3^2}\right)=\frac{z_1}{d_1^2}+\frac{z_2}{d_2^2}+\frac{z_3}{d_3^2} \quad (3)$$

$$\sigma_0^2(a+b+c)^2=\sigma^2_{surface}\left(a^2+b^2+c^2\right) \quad (4)$$

That is to say:

$$\sigma_0=\sigma_{surface}\frac{\sqrt{\left(a^2+b^2+c^2\right)}}{(a+b+c)} \quad (5)$$

A Monte Carlo numerical simulation has been conducted to resolve the last equation, randomly varying the values of the interpolation weights and the number of closest neighbours over 5000 runs. The results have presented a very good agreement (R^2 = 0.9972) to a potential expression given by:

$$\sigma^2_{gridding}=1.0318M^{-0.8646}\sigma^2_{surface} \quad (6)$$

Where M is the number of closest neighbours employed for the IDW interpolation process and $\sigma_{surface}$ is the surface error, given by the sum of two components:

$$\sigma^2_{surface}=\sigma^2_{SDE}+\sigma^2_{IL} \quad (7)$$

Being σ_{IL} the empirical component of the model which refers to what is known as "Information Loss". This is the error due purely to modelling the continuous terrain surface from only a discrete number of points. Then the final model formulation is given by:

$$\sigma^2_{gridding}=1.0318M^{-0.8646}(\sigma^2_{SDE}+\sigma^2_{IL}) \quad (8)$$

Where $\sigma_{gridding}$ is the gridding error across the whole surface and σ_{IL} is the information loss. Information loss was empirically modelled by means of a calibration process described subsequently, which yielded the following expression:

$$\sigma^2_{IL}=0.275847Slope^{0.97307}\mathrm{N}^{-0.49987} \quad (8)$$

Where *Slope* is the average terrain slope (dimensionless expression) and N is the LiDAR ground resolution (points/m^2). Therefore the empirical component regarding information loss embraces two variables directly implied in the final gridding error: terrain complexity and original sampling density.

Referring back to equation 2, the final model formulation can now be written as:

$$\sigma^2_{total}=1.0318M^{-0.8646}(\sigma^2_{SDE}+\sigma^2_{\mathrm{IL}})+\sigma^2_{\mathrm{SDE}} \quad (9)$$

3. STUDY SITES AND DATA SETS

3.1. Data sets used for model calibration

For model calibration 29 study sites, 200 x 200 m in size, belonging to different areas around Almeria province, south-east Spain, were acquired by means of stereo-photogrammetric methods. The range of slopes and geomorphologic conditions of those 29 study sites can be considered as very diverse (from 3% up to 82% average slope) and so they represented an excellent data source for calibrating the empirical component of the model.

The DEM of each topographic surface was obtained automatically by stereo-image matching and subsequent manual editing. In this way seven DEMs with a grid spacing of 2 m were obtained. The grid points were on UTM map projection (zone 30 North; European Datum 1950) and elevation data were stored as orthometric heights.

The different sampling densities used to calibrate the empirical component of the model were extracted from each original grid DEM by stratified random sampling (four by four sampling quadrants), that guaranteed a homogenous distribution of the sampled data over the whole working area. Each sampling density tested was composed of four replicates randomly extracted and ranged from 0.25

point/m^2 (2 m average grid spacing) to 0.0008 points/m^2 (35 m average grid spacing). Residuals, or differences between original and interpolated DEM, were computed by the true validation method over a sample of 169 check points previously extracted by random sampling from the original data sets. The information loss, that is the response or dependent variable in regression analysis for calibrating the empirical component, was hence computed as the standard deviation of those residuals.

3.2. Data sets used for model validation

The developed methodology was validated against two different LiDAR data sets.

The first data set corresponds to an Ordnance Survey (OS) LiDAR survey carried out over a region of Bristol in the UK.

Data were captured in August 2006 with a Riegl Q560 sensor presenting an original ground spacing of between 0.5 to 1 points/m^2 over flat to hilly terrain (approximately 11% average slope). Furthermore, a set of 49 ground check points, natural features distributed throughout the study area (Figure 2 and Figure 3), were surveyed by OS using differential GPS methods, enabling the assessment of the LiDAR survey vertical accuracy. Different LiDAR point densities were acquired from the original LiDAR data by a thinning or decimation process carried out with Terrascan™ software using the central point algorithm. This yielded several average grid spaced DEMs of 4.4 m, 5.3 m, 7 m, 8.4 m, 11.1 m, 13.2 m, 16.9 m and 23.5 m.

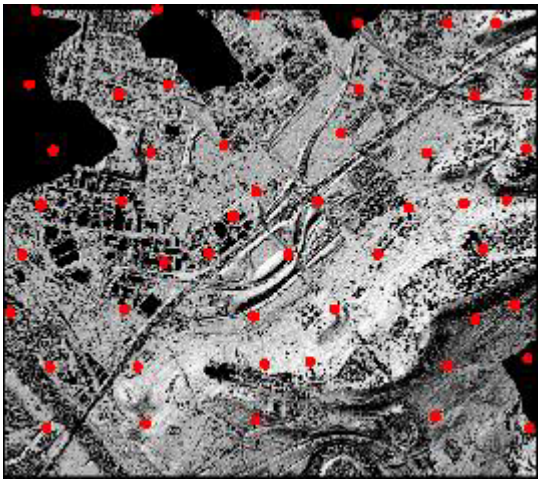

Fig. 2. Aerial view corresponding to the validation area of Bristol, UK, depicting the GPS check point distribution. Reproduced with the permission of Ordnance Survey.

Fig. 3. Ground check point location in the Bristol area (marked cross in the picture). Reproduced with the permission of Ordnance Survey.

The second data set corresponds to an area located at Gador mountain range, south of Almeria province, Spain. LiDAR data was collected in a survey carried out in September 2007 using a Leica ALS50-II scanning system with a flight altitude of 1500 m, providing an approximate resolution of around 0.5 points/m^2. In this case a high resolution image data set (0.2 m ground pixel size) was also obtained with a digital camera, an Intergraph DMC (RGB plus IR).

Last return raw data were filtered to segment the ground surface from vegetation, buildings and any gross errors embedded in the general point cloud. The filtering algorithm developed by Axelsson [11] and implemented in Terrascan™ software was used to segment ground points by means of a progressive TIN densification method where the surface was allowed to fluctuate within certain values. The digital imagery was utilised to help filter non-terrain objects and bare earth terrain.

A number of 15 squared subsets, 286 x 286 m in size (approximately 1.44 m grid spacing), were extracted from the Gador data set presenting a range of average slope between 20% and 61%. Eight LiDAR sampling densities were generated from every one of the original subsets by means of a stratified random sampling. Each sampling density counted on four randomly extracted replicates to incorporate sample point variability, obtaining LiDAR ground resolutions ranging from 0.23 points/m^2 to 0.0008 points/m^2. Again residuals were computed by the true validation method over a sample of 150 check points previously extracted by random sampling from the original subsets.

4. RESULTS AND DISCUSSION

4.1. Model calibration

The empirical component of the model, or information loss in equation 8, showed a good fit to the experimental data, with a R^2 regression coefficient of 0.9856 ($p<0.001$) (Figure 4). It must be highlighted that the standard deviation (Sd) observed and predicted in Figure 4 refers to the term σ_{IL} in equation 8 (information loss). Summing up, the DEM information loss grows almost linearly with increasing slope (rugged terrain) and presents a non-linear inverse relationship regarding LiDAR ground sampling density. That non-linear relationship allows pointing out that a decrease in LiDAR ground resolution only provokes a significant increase of DEM error when we are situated within the bounds corresponding to low LiDAR post-spacing values. Furthermore the break-point location (LiDAR ground sampling density above which practically no gridding error is propagated to the interpolated DEM) can vary depending on the average terrain slope.

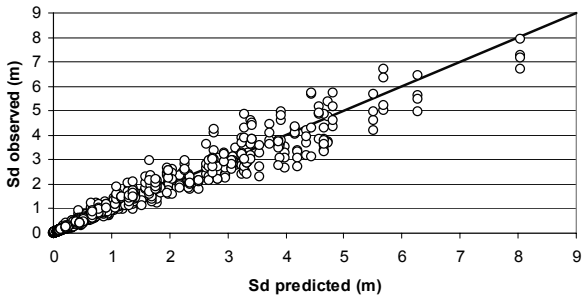

Fig. 4. Results corresponding to the calibration phase of the empirical component "information loss" (see equation 8). R^2 = 0.9856 ($p<0.001$).

Figure 5 shows graphically the aforementioned behaviour for different type of terrain slopes (from 20% up to 100%) according to equation 9. The interpolation method employed was IDW with the local support of the five closest neighbours. It is worth noting that the total error in LiDAR-derived DEMs turns out not to be very sensitive to change in LiDAR nominal post-spacing when the surveyed area presents low average slope. Nevertheless, the influence of LiDAR resolution becomes relevant when terrain complexity increases, and it is therefore recommended to increase LiDAR point density in order to reduce the final total error.

This finding could be deemed as a plausible explanation to the experimental results recently published in [4], where they describe an intriguing absence of a significant pattern relating error in DEM accuracy and different LiDAR post-spacing. The likely explanation is that the terrain where they developed the research presented an average slope ranging between 2% and 5%, a much too low slope to detect significant effects on DEM accuracy due to LiDAR point density variation.

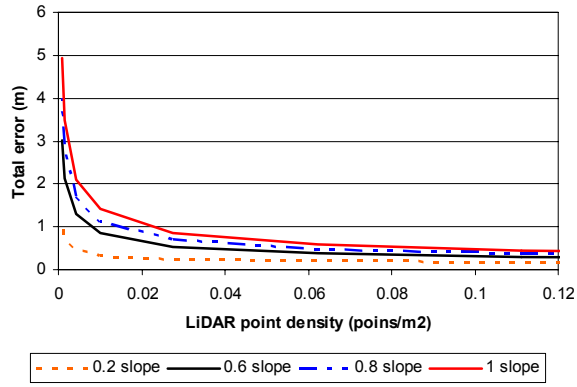

Fig. 5. Graphical representation of the behaviour of the proposed model for SDE = 0.15 m.

4.2. Model validation

The full model developed and calibrated throughout the last sections was validated over the data sets described in Section 3.

Validation over the Bristol data set provided the results depicted in Figure 6. The sample data error (SDE) computed in the Bristol area was 0.124 m and the average slope throughout the whole area was approximately 11%. Starting from the eight previously extracted LiDAR samplings, the interpolation by means of IDW method (five closest neighbours) allowed eight dense DEMs to be obtained (0.5 points/m^2) and their accuracy assessed by means of the GPS check points (observed error).

As can be seen in Figure 6, the results offered by the developed model fit reasonably well to the observed errors, reproducing quite accurately the experimental relationship plotted against total error and LiDAR point density.

Regarding the validation carried out over the Gador data set, it should be underlined that it was only applied to the empirical term determining the so-called information loss, because of the lack of an appropriate set of highly accurate GPS check points when computing the total error. The results are presented in Figure 7, showing a good agreement between predicted and observed errors (R^2 = 0.9272). Given the exhaustive

number of subsets tested and the careful methods used to filter the LiDAR raw data, these results may be deemed as highly significant to validate the empirical component.

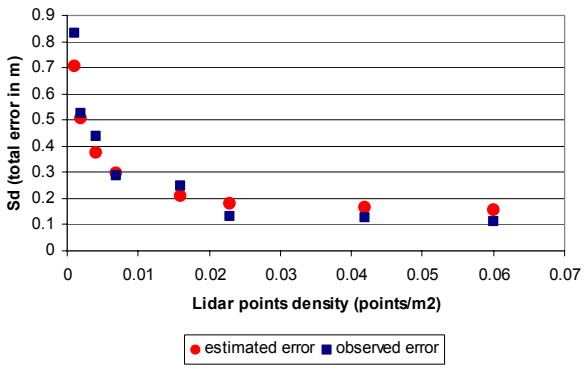

Fig. 6. Validation results corresponding to the Bristol data set. Sd is the standard deviation of residuals at GPS check points for every LiDAR point density tested.

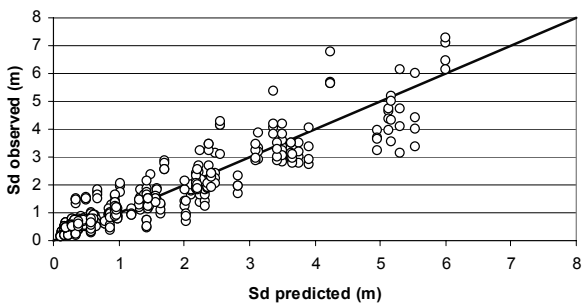

Fig. 7. Validation results corresponding to the Gador data set. Error computed for 15 subsets, eight LiDAR sampling densities and four replicates for every one.

5. CONCLUSIONS

Very little work has been done to determine the minimum data requirements for specific applications of DEMs, although there is an increasing tendency to collect larger volumes of elevation data. In the majority of cases it is preferable to have an optimised DEM adapted to user needs rather than to have a vast amount of data, which will be more difficult to handle. In the particular case of LiDAR data, it must be emphasized that higher LiDAR resolutions generally require an increment in the overall survey costs (e.g. sensor with a higher pulse rate, lower altitude over-flight, narrower scan angle, and so more flight-lines to cover the same area).

In this sense, we have developed a hybrid theoretical-empirical model for modelling the error of LiDAR-derived Digital Elevation Models (DEM) under non-open terrain. Both terrain slope and sampling density were incorporated in the empirical component through the calibration phase, resulting in a very good agreement between predicted and observed data (R^2 = 0.9856; $p < 0.001$). Regarding the validation results, the Bristol observed vertical errors, corresponding to different LiDAR point densities, offered a reasonably good fit to predicted errors. Similarly promising results were achieved in the more rugged morphology of the Gador mountain range data set.

Despite the fact that results to date should be regarded as preliminary, the findings presented in this paper could be used as a guide for the selection of appropriate operational parameters (essentially point density in order to optimize survey costs as a function of terrain complexity), in projects related to LiDAR survey in non-open terrain, for instance those projects dealing with forestry applications.

6. ACKNOWLEDGMENTS

The authors are grateful to Spanish Government for financing the first author's fellowship at Newcastle University through the Spanish Research Mobility Programme "Estancias de profesores e investigadores españoles en centros de enseñanza superior e investigacion extranjeros". Part of this research has been made through the project called "Integración y análisis exploratorio de datos geoespaciales multifuente para el seguimiento y modelado de la evolución y vulnerabilidad de áreas costeras (Proyecto de Excelencia P08-RNM-03575) financed by Consejería de Innovación, Ciencia y Empresa de la Junta de Andalucía.

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